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THE MATHEMATICS TEACHER

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ON THE CURRICULUM OF MATHEMATICS.

By ISAAC J. SCHWATT.

(*Continued from p. 177.*)

The student must thoroughly understand the meaning and the philosophy, so to speak, of each mathematical concept presented to him. It takes a long time before he familiarizes himself so thoroughly with the conceptions of any mathematical subject so that he gets their significance, meaning and spirit, and the ability and facility to apply them readily. Few students have a clear understanding of the quantitative meaning and significance of the theorems of proportion, such as: "A line drawn parallel to a side of a triangle divides the other sides proportionally," or, "Similar triangles are to each other as the squares of the homologous sides." In all my experience I have not received from a pupil a satisfactory explanation of the truth that one divided by infinity is equal to zero, a conception used in secondary mathematics. The same is true of college mathematics. Few students who have studied Analytical Geometry are able to give the true meaning, for instance, of the equation of the straight line, $y = sx + m$, *i. e.*, that the ordinate of any point of the straight line is m greater than s times the abscissa of the point. These instances may be multiplied to quite an extent.

In arithmetic the pupil, as a rule, is unable to understand the reasons for every step taken. He performs most of the operations mechanically. When he is more mature the fundamental operations must again be taken up with the view of gaining clear ideas of the principles on which they are based. As I have already stated, the acquiring of knowledge and of clear ideas is a matter of evolution. I doubt whether we ever gain at once a perfectly clear understanding of an idea, especially if it is a little difficult. It has to grow on us, so to speak. The oftener we reflect and the more we meditate upon it, the clearer it will become to us.

Every idea presented to the student must be adapted to the state of his mental development. The amount of subject matter covered must be such as to make it possible for him without overtaxing his strength to acquire a thorough knowledge and a perfectly clear understanding of every point of it. There must be a fair distribution of the work done in each of the classes in the school and college. The work must be graded in such a manner that each succeeding year the amount covered is a little larger, or the subject taught a little more difficult than in the preceding year. Neither teacher nor pupil should have reason to state that a certain term, unless it is the last term, is the hardest throughout the entire course, or the easiest, unless it is the first term.

The student must not be permitted to do any work by empiric rules. There is, for instance, a pernicious practice of solving quadratic equations by a formula. This mode of effecting the solution does not take any less time than by logical steps. If the student forgets the formula and is not able to derive it, he is helpless. Nothing should be taught in the school or college which savors of the mechanical.

Some mathematical subjects are taught in the high school with little regard to the mental training which they are capable of yielding. Take, for instance, the subject of trigonometry. Most of the instruction in this subject consists in the solution of triangles which involve operations with logarithms. These the student performs, as a rule, in mechanical manner already induced by the use of the logarithmic table. The only occupation in which the trigonometrical solution of triangles may be necessary are surveying and astronomy. It is not to be denied that logarithmic computation develops the qualities of neatness, of being orderly. But these qualities are much more common than the quality which it is the principal purpose of the school to develop, namely, power of mind. It is comparatively easy to secure the services of people who are able to do routine work with neatness and accuracy. But it is often difficult to find a person to fill a position in which mental power, power of discrimination and other qualities are necessary and indispensable.

There ought to be a difference in the kind of mathematics taught those who study the subject for cultural purposes only, and those who have to apply it. The latter must acquire such facility in the operations of algebra and trigonometry as will enable them to perform with ease the operations of calculus, a facility which can be gained only by long and continued practice. While such facility is indispensable to those who may have to use mathematics as a tool, it is not necessary for those who study mathematics for cultural purposes only. The cultural benefit from working examples is not as great as that obtained from a thorough understanding of principles.

The ideas of infinitesimal calculus are of great cultural value. In my opinion, every one who has received a higher education should be familiar with this powerful instrument of the mind. The study of calculus ought to be obligatory in any liberal course. If but one half of the year of two hours a week can be given to it and the work is thoroughly done, it will be sufficient to give the student an idea of the subject and add to his mental powers. I have often been asked by intelligent people whether I could give them in a few words an understanding of what calculus means.

During the time allotted to the study of the mathematical subjects as ordinarily given in the secondary schools, it is not possible for the average student to gain perfectly clear ideas of each of the topics taught; it is best therefore to confine the student to a few topics which he will need in the study of calculus. Very little plane geometry is necessary in the study of the higher branches of mathematics. From trigonometry the student must have perfect facility in the reduction of trigonometrical expressions. The solution of triangles and trigonometrical equations which are performed in a more or less mechanical manner are not essential. Of analytical geometry only the equations of the conics and an ability to plot the curve corresponding to a given function are required. A complete knowledge of the graph of a function can only be obtained by means of calculus.

The mathematics which the average engineer uses in the practice of his profession, as a rule, is very simple. But if the engineer has to do creative work, or to carry out a project which is outside of the ordinary, he is in need of a great deal of mathematical knowledge and skill, sometimes a knowledge of the most advanced branches of the subject. This he cannot obtain during the short four years of his college course. The engineer will have to specialize in such mathematical work just as he must specialize for the more difficult engineering problems. In this respect the medical schools which are the oldest professional schools are best organized. The future physician while at school is given the fundamental principles of his science. If he is to pursue a specialty, he must supplement his knowledge after graduation. No physician without specializing will attempt to treat the diseases of organs like the eye or the ear, or of any complicated disease, for that matter.

To do creative work in mathematics requires, besides a thorough familiarity with the subject, peculiar abilities and powers, gifts with which only a few are endowed. This is true of the elementary as well as the advanced subjects. I fear that mathematicians have not sufficiently emphasized this fact. One may be thoroughly familiar with all the theorems necessary to perform a construction in plane geometry, and yet may not have the peculiar ability to combine these in such a manner as to effect the solution. But the average student ought to be able to follow and comprehend the fundamental ideas of the mathematical disciplines and derive the benefit for his mind which the knowledge of these ideas is capable of yielding.

I see, however, no reason why the study of numbers could not be used as a principal tool, as far as the instruction in mathematics is concerned, for developing power of mind, especially with those who will not continue their studies beyond the high school. We sometimes unconsciously attribute to a person who has studied geometry and trigonometry a broader and better education than to one whose knowledge only extends to operations with number. But a knowledge of some of the properties of the triangle and of the relations of the trigonometric functions has no more cultural value than that of the properties of decimal fractions and periodic decimals, and the different tests of divisibility of number, etc. It is better that the number of tools be few but used with skill. It is not the number of books which a person has read that counts, nor the number of subjects which he has studied at school, but the mental benefit which he has derived from that study.

A great deal of mental training may be derived from solving some of the problems ordinarily given in algebra, by arithmetical processes. It is an invaluable exercise for developing power of mind. Some of the operations of algebra, like the simplification and reductions of fractions and surds are important exercises for training in accuracy, in bringing before the mind rules and principles previously learned and applying them instantaneously to the example on hand, adhering to a strict sequence and order of operations. But there is a peculiar benefit to be derived for the mind from solving problems by means of arithmetical processes. The solution of these by means of equations contains little educational value, while their solution by arithmetical processes often requires a great deal of concentration of mind. Take for instance problems like the following: "I have in mind a number of six digits, the last one on the left being one. If I bring this digit to the first place on the right, I shall obtain a number which is three times the number I have in mind. What is the number?" Or: "A grocer has two kinds of coffee, one at 64 cents and the other at 40 cents per pound. How much of each kind must he take to make a mixture worth 55 cents per pound?" Their arithmetical solution is an excellent exercise for the mind.

I know how difficult it is to break with tradition; I know that it is easier to agree and to follow the trodden path than to disagree and break loose from established and well-rooted practices. But progress necessitates a continuous change and an adaptation to changing conditions. Education seems to have met the change by extending the scope of the curriculum and thereby weakening instead of strengthening its effect.

The best material and the most perfect tools in the hands of the unskilled are of little avail. But the skilful even with inferior material and tools will show their mastery. The teacher with high qualifications and strong personality will help even the least able among his students. The teacher must be a thinker if he is to stimulate his pupils to think. He must be safe and sane, and wise and just, if he is to mould and frame the character of the rising generation. The teacher must be constantly improving himself if he is to give to his pupils a thirst for knowledge and a desire for self-improvement.

It is the boast of modern industry that machinery has been perfected to such a degree as to make it possible in some cases to dispense with intelligent guidance by man. But all the modern physical equipment used in education, which, if properly applied, is a great help in accomplishing its purpose, cannot replace the able teacher, nor can the most excellent textbook take the place of the living word.

No child should be during his entire school life under the influence and guidance of a man or woman only. The time is past when the mother alone had to do with the bringing up of the children and the father was considered only the bread winner. It was true then that: " A wise son maketh a glad father; but a foolish son is the heaviness of his mother " (Proverbs 10, 1). It is the coöperation of the parents, both father and mother, which gives a harmonious development to the character of the child. The good influence of both father and mother upon the child is bound to produce a more perfect man or woman, each maintaining the principal qualities of his or her sex. The man will be better fitted for the struggle in life and for his intercourse with both men and women, if he has acquired some of the gentle qualities of his mother and his woman teacher. The same is true of the girl, who in these days cannot lead the secluded life of former days within the walls of the house. She also will be better fitted for life if she has acquired some of the faculties which are characteristic of men.

As I have said in another place, the teacher enjoys more leisure than the followers of any other profession. There is no other occupation or profession in which even the beginner is free from the actual performance of his duties for a period ranging from one fourth to one third of the year. This estimate includes also Saturdays, on which day, as a rule, no sessions are held. To whom much is given, of him much will be expected. It is therefore the duty of the teacher to use his leisure properly, not only in making himself proficient in the studies which he has to impart to his pupils, but also in informing himself of the world and the things in it, that he may be well fitted to influence his pupils for all that is right, noble and just. By example, as well as precept, the teacher must be to the student an inspiration and a stimulus to both moral and mental superiority.

Besides that many persons have assumed the responsibility of parenthood without being well fitted for this great task, and inasmuch that the child spends the greater part of the day under the influence of the teacher, or in preparing his lessons; the teacher therefore, should be especially well fitted to influence him toward all that is good and noble. He ought to be a specialist so to speak, in the two principal functions of education, the moulding of character and the development of the mind. In moral questions and in questions of health, not the majority but the minority, and very often the smallest minority, determines the status of a community and of the world at large. The unintelligent or ill-advised action of one individual may bring misfortune to, and affect the happiness of many.

The development of the moral and intellectual powers of the individual should be the principal aim of education. The school should inculcate in the young right habits, habits of honesty, of sincerity, of justice, of thoroughness, of striving for perfection in all things. These are, after all, a matter of habit. If we acquire the habit of being thorough, of being orderly, of doing things well, we will not be satisfied unless, whatever we do, we do it right. We shall find it is more difficult to break the habit than it was to acquire it. We must inculcate in the young the habit of right action while they are under our immediate influence and control.

We hear a great deal of the necessity for carrying on campaigns of education to enlighten the public on questions relating to their health, to their morals and their comfort, that they may have regard for the interest of others, not only for this generation, but for all generations to come. The latest expression of this is found in the current agitation for the conservation of natural resources, etc. If we educate the young to have a receptive mind, a willing heart, an open ear, and a susceptible intellect, future generations will have less need for educating the public at large. It is difficult to get a hearing from those who are no longer responsible to us. The most instructive teaching and preaching I have heard was attended, as a rule, by such as needed it the least. Those who should have listened were not in the audience.

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Let me but do my work from day to day,
In field or forest, at desk or loom,
In roaring market place or tranquil room,
Let me but find it in my heart to say,
When vagrant wishes beckon me astray,
This is my work, my blessing, not my doom.

Of all who live, I am the one by whom
This work can best be done in my own way.

Then shall I see it not too great or small
To suit my spirit and to prove my powers;
Then shall I cheerfully greet the laboring hours,
And cheerfully turn, when the long shadows fall,
At eventide, to play, to love, and rest,
Because I know for me my work is best.

—*H. Van Dyke.*